**Vector spaces**

* add two vectors, multiply vectors by scalars(linear combinations still in the space)
* A real vector space is a set of vectors **together with rules** for vector addition and multiplication by real numbers.
* vector can be
  + infinite-dimensional vector
  + matrix
  + function that are defined on a fixed interval
* vector：dimensions, components

**subspace** of a vector space

* definition: A **subspace** of a vector space is a **nonempty subset** that satisfies the requirements for a vector space: **Linear combinations stay in the subspace**.
* **zero vector** will belong to **every subspace**.
* **smallest subspace** Z comtains only **one vector** 
  + the **zero vector**(**zero-dimensional space**)
  + **empty set** is not allowed
* **largest subspace** L is the original space **itself**.
* the subspace of R3:
  + R3 **itself**
  + **any plane** through the origin
  + **any line** through the origin
  + the **origin**(zero vector)
* example of subspaces(R2)
  + 子集:第一象限**不是子空间**(乘以负数标量后=>第三象限)
    - 一个空间需要允许正负
  + 子集:第一象限+第三象限**不是子空间**(线性组合后会到达2,4象限: 不是子集本身)
  + the smallest subspace containing first quadrant is the whole space R2
* example of subspaces(matrix)
  + 子集: lower triangular matrices**是子空间**(相当于去掉了几个维数(0s))
  + 子集: symmetric matrices**是子空间** 相当于
    - 某几维以**一定倍数捆绑后，共同增减**(斜率)(因为这两个维相关，相当于降低了一个维度)
    - 也就是为什么做应用题目联立方程组时，如果两个变量有关系，可以降低维度
    - 降低维度的方法：比如二维：固定某个维度的值
      * 1) 使某个维的值固定(x=0, x=1)[产生了垂线]
      * 2) 使两个维的值产生关系[产生了斜线]
      * 满足条件y=nx的直线
      * 思维误区：不是说只有x=0 时才是降维，x=0只是降维的一种特例(垂直)，降维是空间缩小(子空间)，不一定要垂直
  + 对于components的**范围**做限制一般不行
  + 但是几个不同维基上的components的**关系**做限制可行
* the column space of A
  + definition: column space of A consists of all combinations of columns of A.
    - denoted by C(A)
  + the column space comtains all linear combinations of the columns of A
    - it’s a subspace of Rm
  + the system Ax = b is solvable if and only if the vector b can be expressed as a combination of the columns of A. Then b is in the column space.
  + 让b落在这个“子空间”里，就有解；特别的方法是
    - 让b落在这个“子空间”的“子空间”里, x=0;y=0;x=y=0
  + C(I) is the whole of R5 ,the five columns of I can combine to produce any five-dimensional vector b
  + any 5 by 5 matrix that is nonsingular will have the whole of R5 as its column space
  + every b is in C(A) for a nonsingular matrix
    - C(A) can be somewhere between the zero space and the whole space Rm
* the nullspace of A
  + the nullspace of a matrix consists of all vectors x such that Ax=0.
  + denoted by N(A). it’s a subspace of Rn as the column space was a subpace of Rm
  + both requirements fail if the right-had side is not zero(since it doesn’t through origin)
  + the nullspace contains only the vector(0,0)
    - if column vectors are independent columns
  + the nullspace can be higher dimensional space
    - if some column is combinations of the others.
* idea for any system Ax=0
  + to find C(A) and N(A)
    - C(A):all attainable right-hand sides b
      * vectors b are in the column space
    - N(A): all solutions to Ax=0
      * the vectors x are in the nullspace
* 1. x + y = y + x.
* 2. x + (y + z) = (x + y) + z.
* 3. There is a unique “zero vector” such that x + 0 = x for all x.
* 4. For each x there is a unique vector −x such that x + (−x) = 0.
* 5. 1x = x.
* 6. (c 1 c 2 )x = c 1 (c 2 x).
* 7. c(x + y) = cx + cy.
* 8. (c 1 + c 2 )x = c 1 x + c 2 x.
* example
  + suppose A is the 5 by 5 identity matrix.
  + C(I) is the whole of R5
  + the five columns of I can combine to produce any five-dimensional vector b
  + any 5 by 5 matrix that is nonsingular will have the whole of R5 as its column space. for such a matrix we can solve Ax = b by gaussian elimination
  + there are five pivots
  + therefore every b is in C(A) for a nonsingular matrix.